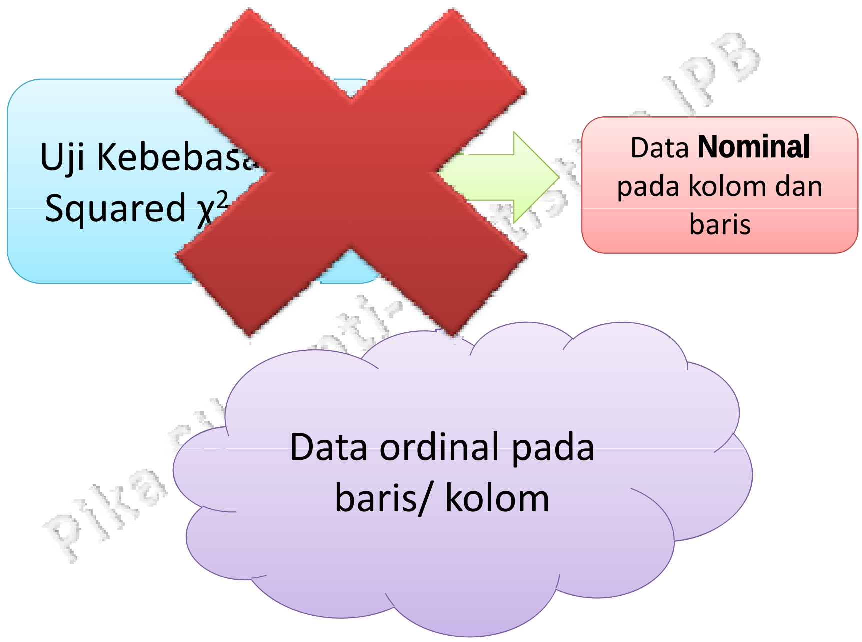
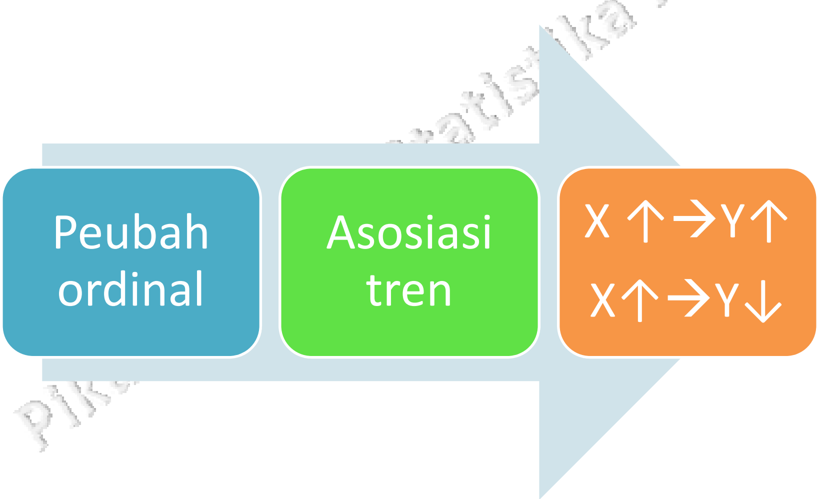
# Tabel Kontingensi 2x2 (4)

Uji Kebebasan untuk Data Ordinal

Uji Eksak untuk Ukuran Contoh Kecil



## Uji Kecenderungan Linier




## Uji Kecenderungan Linier

- *u1 ≤ u2 ≤ · · ·≤ ui* → *skor baris, dan*
- *v1 ≤ v2 ≤ · · · ≤ vj* → skor kolom
- Urutan skor sama dengan level kategori
- *Dengan* $\bar{u} = \sum_i u_i p_{i+}$ *dan* $\bar{v} = \sum_j v_j p_{+j}$
- *Korelasi*

$$r = \frac{\sum_{i,j}(u_i - \bar{u})(v_j - \bar{v})p_{ij}}{\sqrt{\left[\sum_i (u_i - \bar{u})^2 p_{i+}\right]\left[\sum_j (v_j - \bar{v})^2 p_{+j}\right]}}$$

- Hipotesis

  $H_0$: *Peubah baris dan kolom saling bebas vs*

  $H_a$: $\rho \neq 0$,
- Statistik Uji : $M^2 = (n - 1)r^2$
- Untuk nilai n yang besar, $M^2$ mendekati sebaran *chi-squared* dengan db= 1.
- $M = \sqrt{(n - 1)}r$, *mengikuti sebaran normal baku*. Pada hipotesis alternatif satu arah, seperti $H_a : \rho > 0$.
- *Seperti pada* $\chi^2$ *dan* $G^2$, $M^2$ *pun tidak memperhatikan mana peubah respon/penjelas*

## Ilustrasi: **Alcohol Use and Infant Malformation**

- prospective study of maternal drinking and congenital malformations.
- After the first 3 months of pregnancy, the women in the sample completeda questionnaire about alcohol consumption.
- Following childbirth, observations were recorded on the presence or absence of congenital sex organ malformations.
- Alcoholconsumption, measured as average number of drinks per day, is an explanatory variable with ordered categories.
- Malformation, the response variable, is nominal.
- $n = 32,574$

**Table 2.7. Infant Malformation and Mother's Alcohol Consumption**

| Alcohol Consumption | Malformation Absent | Malformation Present | Total | Percentage Present | Standardized Residual |
|---|---|---|---|---|---|
| 0 | 17,066 | 48 | 17,114 | 0.28 | −0.18 |
| <1 | 14,464 | 38 | 14,502 | 0.26 | −0.71 |
| 1–2 | 788 | 5 | 793 | 0.63 | 1.84 |
| 3–5 | 126 | 1 | 127 | 0.79 | 1.06 |
| ≥6 | 37 | 1 | 38 | 2.63 | 2.71 |

*Source*: B. I. Graubard and E. L. Korn, *Biometrics*, **43**: 471–476, 1987. Reprinted with permission from the Biometric Society.

*df = 4, $G^2$ = 6.2 (P = 0.19)*

*df = 4, $X^2$ = 12.1 (P = 0.02)*

# Dengan uji kecenderungan linier

- *v1 = 0, v2 = 0.5, v3 = 1.5, v4 = 4.0,v5 = 7.0, skor terakhir ditentukan secara sembarang.*
- *r = 0.0142.*
- Statistik Uji *$M^2$ = (32,573)(0.0142)2 = 6.6 memiliki P-value = 0.01, berarti cukup bukti mengatakan bahwa ada korelasi* (nonzero correlation).
- Statistik normal baku *M = 2.56 memiliki P = 0.005 untuk Ha: ρ > 0.*

# Syntax SAS untuk menghitung $M^2$

```
DATA alcohol;
INPUT item1 $ item2 $ row col count;
DATALINES;
strongagree strongagree 1 1 97
strongagree agree 1 2 96
... ...
strongdis strongdis 4 5 2
;
/*For the TABLES command, use the numeric variables that
contain the row and column scores.*/
PROC FREQ;
TABLES row*col / chisq measures;
```

- membaca output output:
  - "Mantel-Haenszel Chi-Square" adalah $M^2$ (untuk skor dengan jarak yang sama).
  - "Pearson correlation" adalah r.



# Bagaimana menentukan skor yang tepat?

| Alkohol consumption | Skor |
|---|---|
| 0 | 10 |
| <1 | 20 |
| 1-2 | 30 |
| 3-5 | 40 |
| ≥6 | 50 |

→ **$M^2 = 1.83$, (P = 0.18)**

# Alternatif → Midrank sebagai skor

| Alcohol cons | Malformation | | Total | kum | Midrank | |
|---|---|---|---|---|---|---|
| | | | | | 17114)/2= | 8557,5 |
| | | | | | *1,616)/2=* | 24,3655 |
| 1 | | | | | +32409)/2= | 32013 |
| 3-5 | 126 | 1 | 127 | 32536 | (3 32536)/2= | 32473 |
| ≥6 | 37 | 1 | | | 2537+32574)/2= | 32555,5 |

Konsekwensinya adalah bahwa skema penilaian ini memperlakukan tingkat konsumsi alkohol 1-2 (kategori 3) lebih dekat dengan tingkat konsumsi ≥6 (kategori 5) daripada tingkat konsumsi 0 (kategori 1).

**$M^2 = 0,35$, ($P = 0.55$)**

# Sytntax Sas untuk midranks

```
PROC FREQ;
TABLES row*col / cmh1 scores=ridits;
```

# Ilustrasi SAS data alcohol

```
data alcohol;
input dose $ malformation $ row col count;
datalines;
0 absent 1 1 17066
0 present 1 2 48
<1 absent 2 1 14464
<1 present 2 2 38
1-2 absent 3 1 788
1-2 present 3 2 5
3-5 absent 4 1 126
3-5 present 4 2 1
>=6 absent 5 1 37
>=6 present 5 2 1
;
PROC FREQ;
TABLES row*col / nopercent nocol norow chisq measures cmh1
  scores=ridits;
weight count;
run;
```



# Output

| Statistic | DF | Value | Prob |
|---|---|---|---|
| Chi-Square | 4 | 12.0821 | 0.0168 |
| Likelihood Ratio Chi-Square | 4 | 6.2020 | 0.1846 |
| MH Chi-Square (Ridit Scores) | 1 | 0.3514 | 0.5533 |
| Phi Coefficient | | 0.0193 | |
| Contingency Coefficient | | 0.0193 | |
| Cramer's V | | 0.0193 | |

WARNING: 30% of the cells have expected counts less than 5. Chi-Square may not be a valid test.

- Statistik Uji $M^2$ memperlakukan kedua klasifikasi sebagai ordinal. Ketika satu variabel (misalnya X) adalah nominal tetapi hanya memiliki dua kategori, kita masih bisa menggunakannya.
- Ketika X adalah nominal dengan lebih dari dua kategori, uji ini tidak lagi sesuai untuk digunakan.



# Alternatif lain

- gamma
- Kendall's tau-b
- *Cochran–Armitage trend test* → Dibahas pada BAB 6

# Tabel Kontingensi lain

Caesarean section and shoe size

| C section | $< 4$ | 4 | 4.5 | 5 | 5.5 | $\geq 6$ | total |
|---|---|---|---|---|---|---|---|
| Yes | 5 | 7 | 6 | 7 | 8 | 10 | 43 |
| No | 17 | 28 | 36 | 41 | 46 | 140 | 308 |
| Total | 22 | 35 | 42 | 48 | 54 | 150 | 351 |

- Khi-kuadrat tradisional menilai perbedaan laju perbedaan C-section diantara grup ukuran sepatu tetapi tanpa aturan tertentu.
- Ukuran sepatu mungkin menggambarkan ukuran pinggul. Jadi mungkin ada pola hubungan antara C-section dengan ukuran sepatu
- Pengujian hipotesis yang lebih spesifik → kuasa pengujian yang lebih tinggi

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# Hipotesis, Frek Harapan, Statistik Uji

Caesarean section and shoe size

| C section | $< 4$ | 4 | 4.5 | 5 | 5.5 | $\geq 6$ | total |
|---|---|---|---|---|---|---|---|
| Yes | 5 | 7 | 6 | 7 | 8 | 10 | 43 |
| No | 17 | 28 | 36 | 41 | 46 | 140 | 308 |
| Total | 22 | 35 | 42 | 48 | 54 | 150 | 351 |
| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $\bar{p}$ |

- Ho: $p_1 = p_2 = \ldots = p_6$. H1: ada kecenderungan linear $p_2=p_1+\Delta$, $p3=p1+2\Delta$, …, $p6=p1+5\Delta$
- Hitung dugaan $p_1$, $p_2$, …, $p_6$ dengan asumsi kecenderungan linear
- Hitung frekuensi harapan jika Ho benar
- Hitung statistik uji:

$$\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{6} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

KAN, 2014

## Frekuensi Harapan

Caesarean section and shoe size

| C section | $< 4$ | 4 | 4.5 | 5 | 5.5 | $\geq 6$ | total |
|---|---|---|---|---|---|---|---|
| Yes | 2.70 | 4.29 | 5.15 | 5.88 | 6.62 | 18.38 | 43 |
| No | 19.30 | 30.71 | 36.85 | 42.12 | 47.38 | 131.62 | 308 |
| Total | 22 | 35 | 42 | 48 | 54 | 150 | 351 |

$$\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{6} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$ ⇨ $(5-2.70)^2/2.70 + \ldots + (140-131.62)^2/131.62$

$\chi^2 = 9.29$ with $df = 5 \rightarrow P$-value$=0.098$

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## Alternatif 1: Uji kecenderungan linear

Caesarean section and shoe size

| C section | $< 4$ | 4 | 4.5 | 5 | 5.5 | $\geq 6$ | total |
|---|---|---|---|---|---|---|---|
| Yes | 2.70 | 4.29 | 5.15 | 5.88 | 6.62 | 18.38 | 43 |
| No | 19.30 | 30.71 | 36.85 | 42.12 | 47.38 | 131.62 | 308 |
| Total | 22 | 35 | 42 | 48 | 54 | 150 | 351 |

$\chi^2 = 8.02$ with $df = 1 \rightarrow P$-value$=0.005$

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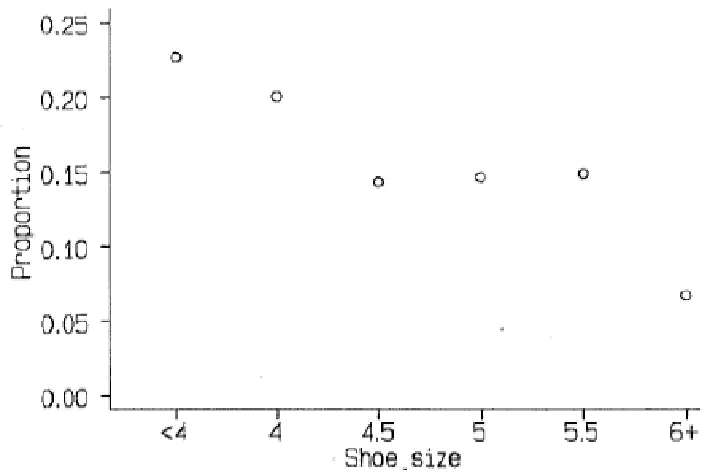
## Diagram pencar






## Alternatif 2: Regresi linear

```
The regression equation is
p = 0.299 - 0.0292 Sepatu


Predictor        Coef     SE Coef       T       P
Constant      0.29872     0.02085   14.32   0.000
Sepatu      -0.029205    0.004016   -7.27   0.002


S = 0.0164173   R-Sq = 93.0%   R-Sq(adj) = 91.2%
```

Dugaan nilai $p$ berdasarkan regresi kecenderungan linear:

$p_1$-duga = 0.2260 $p_4$-duga = 0.1530
$p_2$-duga = 0.1822 $p_5$-duga = 0.1384
$p_3$-duga = 0.1676 $p_6$-duga = 0.0654

**WARNING: 30% of the cells have expected counts less than 5. Chi-Square may not be a valid test.**

- Uji Chi-square tidak valid jika ukuran contoh relatif kecil → lebih dari 25% sel memiliki nilai harapan< 5 → see WARNING under the result of test.
- Saat n kecil, inferensia bisa dilakukan dengan melihat *exact distributions* dibandingkan *large-sample approximations*



# Fisher's Exact Test (Uji Pasti Fisher)

Based on Hypergeometric distribution

$$p(t) = P(n_{11} = t) = \frac{\binom{n_{1+}}{t}\binom{n_{2+}}{n_{+1} - t}}{\binom{n}{n_{+1}}}.$$

Hipotesis nol pada uji pasti fisher adalah kedua peubah (baris dan kolom) saling bebas

# Uji Pasti Fisher (lanjutan)



- Uji pasti Fisher berlaku untuk **semua ukuran contoh** (tidak hanya untuk ukuran contoh kecil)
- Untuk ukuran contoh besar uji ini memerlukan waktu komputasi yang lama. Nilai-p yang dihasilkan akan mendekati nilai-p dari uji khi-kuadrat (chi-squared)
- Uji khi-kuadrat efisien jika ukuran contoh besar

# Tabel 2x2

| | men | women | total |
|---|---|---|---|
| dieting | a | b | a + b |
| not dieting | c | d | c + d |
| totals | a + c | b + d | n |

Rasio odds

$$\hat{\theta} = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

$$p = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}} = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!}$$

# Tahapan Uji Pasti Fisher

1. Susun Hipotesis $H_0$:$p_1$=$p_2$
2. Buat tabel-tabel yang lebih "ekstrim" dengan mengurangi pengamatan terkecilnya tetapi jumlah baris dan kolomnya harus tetap
3. Hitung semua nilai $p_i$ untuk seluruh tabel tersebut
4. Tentukan $p_{hit}=p_1+p_2+p_3+p_4$, dan tolak $H_0$ jika $p_{hit}<\alpha$(uji 1 arah) atau $p_{hit}<\alpha/2$(uji 2 arah)



# Contoh Kasus

Seseorang ingin melihat hubungan antara pola diet seseorang dengan jenis kelamin. Uji pada taraf 5% apakah proporsi jenis kelamin pada yang melakukan diet dan yang tidak diet sama atau tidak

1

| | men | women | total |
|---|---|---|---|
| dieting | 9 | 6 | 15 |
| not dieting | **3** | 4 | 7 |
| totals | 12 | 10 | 22 |

$H_0$:$p_1$=$p_2$ VS $H_1$: $p_1 \neq p_2$

# Buat tabel lebih ekstrim…

3

| | men | women | total |
|---|---|---|---|
| dieting | 10 | 5 | 15 |
| not dieting | **2** | 5 | 7 |
| totals | 12 | 10 | 22 |

2

| | men | women | total |
|---|---|---|---|
| dieting | 11 | 4 | 15 |
| not dieting | **1** | 6 | 7 |
| totals | 12 | 10 | 22 |

4

| | men | women | total |
|---|---|---|---|
| dieting | 12 | 3 | 15 |
| not dieting | **0** | 7 | 7 |
| totals | 12 | 10 | 22 |

# Hitung semua $p_i$..

$$p_1 = \frac{12!10!15!7!}{22!9!3!6!4!} = 0.270897$$

$$p_2 = \frac{12!10!15!7!}{22!10!2!5!5!} = 0.09752$$

$$p_3 = \frac{12!10!15!7!}{22!11!1!4!6!} = 0.014776$$

$$p_4 = \frac{12!10!15!7!}{22!0!3!7!2!} = 0.0007036307$$

# $P_{hit}$ dan keputusan…

**$P_{hit}$=0.270897+0.09752+0.014776+0.0007036307**
**=0.3839**

**Karena $P_{hit}$>0.025, maka terima $H_0$**

**→ Belum cukup bukti mengatakan bahwa proporsi jenis kelamin pada yang melakukan diet dan yang tidak diet berbeda**



# Ilustrasi

- To illustrate this test in his 1935 book, *The Design of Experiments, Fisher described* the following experiment: When drinking tea, a colleague of Fisher's at Rothamsted Experiment Station near London claimed she could distinguish whether milk or tea was added to the cup first.
- To test her claim, Fisher designed an experiment in which she tasted eight cups of tea. Four cups had milk added first, and the other four had tea added first.
- She was told there were four cups of each type and she should try to select the four that had milk added first.
- The cups were presented to her in random order.

- The null hypothesis *H0: ϑ = 1* for Fisher's exact test states that her guess was independent of the actual order of pouring.
- The alternative hypothesis that reflects her claim, predicting a positive association between true order of pouring and her guess, is *Ha: ϑ > 1*

**Table 2.8. Fisher's Tea Tasting Experiment**

| | Guess Poured First | | |
|---|---|---|---|
| Poured First | Milk | Tea | Total |
| Milk | 3 | 1 | 4 |
| Tea | 1 | 3 | 4 |
| Total | 4 | 4 | |

*Hipotesis*

*H0: ϑ = 1 vs Ha: ϑ > 1*

| Poured | |
|---|---|
| | milk |
| Milk | 0 |
| tea | 0 |
| Total | 4 |

P = P(3) + P(4) = 0.243

Kesimpulan:

Kapena p> 0,05 berarti belum cukup bukti untuk menolak H0. Tidak ada asosiasi antara urutan menuang dengan tebakan

$$P(3) = \frac{\binom{4}{3}\binom{4}{1}}{\binom{8}{4}} = \frac{[4!/(3!)(1!)][4!/(1!)(3!)]}{[8!/(4!)(4!)]} = \frac{16}{70} = 0.229$$

$$P(4) = \binom{4}{4}\binom{4}{0}\Big/\binom{8}{4} = 1/70 = 0.014$$

# Syntax SAS

```
data tea;
input poured $ guess $ count;
datalines;
milk milk 3
milk tea 1
tea milk 1
tea tea 3
;
proc freq data=tea;
tables poured*guess/ nopercent nocol norow chisq;
weight count;
exact pchi chisq or;
run;
```



| Fisher's Exact Test | |
|---|---|
| Cell (1,1) Frequency (F) | 3 |
| Left-sided Pr <= F | 0.9857 |
| Right-sided Pr >= F | 0.2429 |
| | |
| Table Probability (P) | 0.2286 |
| Two-sided Pr <= P | 0.4857 |

| Odds Ratio (Case-Control Study) | |
|---|---|
| Odds Ratio | 9.0000 |
| | |
| Asymptotic Conf Limits | |
| 95% Lower Conf Limit | 0.3666 |
| 95% Upper Conf Limit | 220.9270 |
| | |
| Exact Conf Limits | |
| [illegible] | 0.2117 |
| 95% Upper Conf Limit | 626.2435 |

Selang sangat lebar, karena jumlah n yang sangat kecil